

$$1) H(j\omega) = \frac{100 j\omega}{(j\omega + 1) \left((j\omega)^2 + \underbrace{10 j\omega}_{2\zeta\omega_n} + \underbrace{100}_{\omega_n^2} \right)}$$

$$\begin{aligned} \omega_n^2 &= 100 \\ \omega_n &= 10 \\ 2\zeta\omega_n &= 10 \\ \zeta &= \frac{1}{2} \end{aligned}$$

$$= \frac{10 j\omega}{\left[\left(\frac{j\omega}{0.1} \right) + 1 \right] \left[\left(\frac{j\omega}{10} \right)^2 + \left(\frac{j\omega}{10} \right) + 1 \right]}$$

\uparrow Break at 0.1 \uparrow Break at 10.0

- 2) +20 dB/dec break at $\omega = 0.1$ (first order)
 -40 dB/dec break at $\omega = 10$ (second order)
 20 dB peak implies a damping ratio of .05

$$H(j\omega) = \frac{1}{j\omega} \frac{\left(\frac{j\omega}{0.1} + 1 \right)}{\left(\left(\frac{j\omega}{10} \right)^2 + .01 \left(\frac{j\omega}{10} \right) + 1 \right)}$$

$$= \frac{1000 (j\omega + 1)}{(j\omega)^2 + 1.0 j\omega + 100}$$

$$3) x(t) = \cos t - \sin t$$

$$\mathcal{L}[\sin t] = \frac{1}{s^2+1}$$

$$\frac{d}{dt} \sin t = \cos t$$

$$\begin{aligned} \mathcal{L}[\cos t] &= \mathcal{L}\left[\frac{d}{dt} \sin t\right] = s \left[\frac{1}{s^2+1} \right] - \sin t \Big|_{t=0} \\ &= \frac{s}{s^2+1} \end{aligned}$$

$$\therefore \underline{X}(s) = \frac{s}{s^2+1} - \frac{1}{s^2+1} = \frac{s-1}{s^2+1} = \frac{s-1}{(s+j)(s-j)}$$

